

# HOLOGRAPHIC DARK ENERGY AND VALIDITY OF THE GENERALIZED SECOND LAW OF THERMODYNAMICS IN THE DGP BRANE WORLD

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In this paper, we investigate the validity of the generalized second law of thermodynamics of the universe in the DGP brane world. The boundary of the universe is assumed to be enclosed by the dynamical apparent horizon or the event horizon. The universe is chosen to be homogeneous and isotropic and the validity of the first law has been assumed here. The matter in the universe is taken in the form of non-interacting two fluid system- one component is the holographic dark energy model and the other component is in the form of dust.

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## I. INTRODUCTION

Astrophysical observations made at the turn of the last century [1] show conclusive evidence for acceleration in the late universe, which is still a challenge for cosmologists. It shows beginning of accelerated expansion in the recent past. It is found that cosmic acceleration is driven by some invisible fluid having its gravitational effect in the very late universe. This unknown fluid has distinguishing feature of violating strong energy condition (SEC) being called dark energy (DE) [2]. Various models have been proposed to solve this problem. A comprehensive review of these models is available in [3].

In the race to investigate a viable cosmological model, satisfying observational constraints and explaining present cosmic acceleration, brane-gravity was introduced and brane-cosmology was developed. A review on brane-gravity and its various applications with special attention to cosmology is available in [4].

A simple and well studied model of brane-gravity (BG) is the Dvali-Gabadadze-Porrati (DGP) braneworld model [5]. In this model our 4-dimensional world is a FRW brane embedded in a 5-dimensional Minkowski bulk. It explains the origin of DE as the gravity on the brane leaking to the bulk at large scale. On the 4-dimensional brane the action of gravity is proportional to  $M_P^2$  whereas in the bulk it is proportional to the corresponding quantity in 5-dimensions. The model is then characterized by a cross over length scale

$$r_c = \frac{M_P^2}{2M_5^2}$$

such that gravity is 4-dimensional theory at scales  $a \ll r_c$  where matter behaves as pressure less dust but gravity *leaks out* into the bulk at scales  $a \gg r_c$  and matter approaches the behaviour of a cosmological constant.

In this conceptual set up, one of the important questions concerns the thermodynamical behaviour of an accelerated expanding universe driven by DE. Motivated by the profound connection between black hole physics and thermodynamics, in recent times there has been some deep thinking on the relation between gravity and thermodynamics. A pioneer work in this respect was done by Jacobson who disclosed that Einstein's gravitational field equation can be derived from the relation between horizon area and entropy together with Clausius relation  $\delta Q = T\delta S$  [6]. Some recent discussion on the connection between gravity and thermodynamics on

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various gravity theories can be found on [7]. Study of thermodynamics in Einstein's gravity was investigated in [8]. Recently this connection between gravity and thermodynamics has been extended to brane world scenarios [9]. In ref [10] it is shown that apparent horizon entropy extracted through connection between gravity and first law of thermodynamics satisfies the generalised second law of thermodynamics (GSLT) in DGP warped brane. In General Relativity (GR) frame work the authors of [11] have shown in contrast to the case of the apparent horizon, both first and second law of thermodynamics breakdown if one considers boundary of the universe to be the event horizon. But so far attempts to address these problems are made using apparent horizon only in brane world scenario. So it is imperative to study GSLT using event horizon as the boundary of the universe in BG set up.

The other way to approach to the problem of DE arises from holographic principle which states that the number of degrees of freedom for a system within a finite region should be finite and is bounded by the area of its boundary. As in ref [12] one obtains holographic energy density as

$$\rho_D = 3c^2 M_P^2 L^{-2}$$

where  $L$  is an IR cut-off in units  $M_P^2 = 1$ . Li shows that [13] if we choose  $L$  as the radius of the event horizon we can get the correct equation of state and get the desired accelerating universe.

It may be noted that in literature, standard DGP model has been generalized to (i) LDGP model by adding a cosmological constant [14], (ii) QDGP model by adding quissence perfect fluid [15], (iii) CDGP by Chaplygin gas [16] (iv) SDGP by a scalar field [17]. In [18] the authors analysed the DGP model by adding HDE.

Often using BG one obtains cosmological surprises. Thus it would be interesting to investigate the Generalized Second Law of thermodynamics (GSLT) using BG based theory.

In a recent paper [19], validity of GSLT has been studied in Einstein's gravity and Gauss Bonnet (GB) gravity by assuming first law of thermodynamics and conditions for validity of GSLT have been obtained. Aim of the present paper is to extend the work of [19] in DGP model of BG.

Here we study the validity of GSLT of the universe bounded by (i) the dynamical apparent horizon (ii) event horizon in the DGP brane world. We assume HDE density  $\rho_\Lambda = 3c^2 M_P^2 L^{-2}$  still holds in the DGP model and consider the evolution of HDE on the brane according to Holographic principle. The matter in the universe is taken in the form of non-interacting two fluid system- one component is the holographic dark energy and the other component is in the form of dust (CDM). At the apparent horizon it is shown that GSLT is always respected regardless of specific form of DE. But in case bounarary is bounded by event horizon GSLT may breakdown in the future universe.

The paper is organized as follows : Section 2 deals with HDE in the DGP brane model while validity of GSLT has been examined for apparent and event horizon in section 3. The paper ends with a conclusion in section 4.

## II. HDE IN THE DGP MODEL

In flat, homogeneous and isotropic brane the Friedmann equation [5] is given by

$$H^2 = \left( \sqrt{\frac{\rho_t}{3}} + \frac{1}{4r_c^2} + \epsilon \frac{1}{2r_c} \right)^2 \quad (2.1)$$

or equivalently

$$H^2 - \epsilon \frac{H}{r_c} = \frac{\rho_t}{3} \quad (2.2)$$

where  $H = \frac{\dot{a}}{a}$  is the Hubble parameter,  $\rho_t$  is the total cosmic fluid energy density and  $r_c = \frac{M_P^2}{2M_5^2}$  is the crossover scale which determines the transition from 4D to 5D behavior and  $\epsilon = \pm 1$ .

(For simplicity we are using  $8\pi G = 1$ )

For  $\epsilon = 1$ , we have standard DGP(+) model which is self accelerating model without any form of dark energy, and effective  $\omega$  is always non phantom. However for  $\epsilon = -1$ , we have DGP(-) model which does not self accelerate but requires dark energy on the brane. It experiences 5D gravitational modifications to its dynamics which effectively screen dark energy.

Here we take  $\rho_t = \rho_m + \rho_D$  where  $\rho_m$  is the energy density of CDM and  $\rho_D$  is the energy density of DE.

The Friedmann eq.(2.2) can be written as

$$H^2 = \frac{1}{3}(\rho_m + \rho_{eff}) \quad (2.3)$$

where  $\rho_{eff}$  is the effective energy density given by

$$\rho_{eff} = \rho_D + \epsilon \frac{3H}{r_c} \quad (2.4)$$

Here we assume that there is no interaction between matter and DE. As the two component matter system is non-interacting so they satisfy energy conservation separately, i.e.

$$\dot{\rho}_m + 3H\rho_m = 0 \quad (2.5)$$

$$\dot{\rho}_D + 3H(\rho_D + p_D) = 0 \quad (2.6)$$

where  $p_D$  is the thermodynamic pressure of DE. Also we have conservation equation for effective energy density[16]

$$\dot{\rho}_{eff} + 3H(1 + \omega_{eff})\rho_{eff} = 0 \quad (2.7)$$

where  $\omega_{eff} = p_{eff}/\rho_{eff}$

Eqs. (2.3) and (2.7) describe the equivalent GR model. Our choice for HDE density is

$$\rho_D = \frac{3c^2}{R_E^2} \quad (2.9)$$

where  $c$  is a constant and  $R_E$  is the future event horizon, given by

$$R_E = a \int_t^\infty \frac{dt}{a} = a \int_a^\infty \frac{da}{Ha^2} \quad (2.10)$$

The Friedmann eq.(2.2) can be rewritten as

$$\frac{H}{H_0} = \sqrt{\Omega_m + \Omega_D + \Omega_{r_c}} + \epsilon \sqrt{\Omega_{r_c}} \quad (2.11)$$

where

$$\Omega_m = \frac{\rho_m}{3H_0^2}, \quad \Omega_D = \frac{\rho_D}{3H_0^2}, \quad \Omega_{r_c} = \frac{1}{4r_c^2 H_0^2} \quad (2.12)$$

are the usual dimensionless density parameters and  $H_0$  is Hubble parameter at redshift  $z = 0$ .

Substituting the value of  $\rho_D$  from (2.9) in eq. (2.10) [18], and then differentiating the resulting equation w.r.t red shift  $z = 1/a - 1$ , we get the evolution of  $\Omega_D$  as

$$\frac{d\Omega_D}{dz} = \frac{2\Omega_D^{3/2}}{c(1+z)} \left( \frac{c}{\sqrt{\Omega_D}} - \frac{1}{\sqrt{\Omega_m + \Omega_D + \Omega_{r_c}} + \epsilon \sqrt{\Omega_{r_c}}} \right) \quad (2.13)$$

From eq(2.2), setting  $z = 0$  we get the initial condition of the above differential equation as

$$\Omega_D(0) = 1 - 2\epsilon \sqrt{\Omega_{r_c}} - \Omega_m(0) \quad (2.13a)$$

We shall solve the above differential equation later on numerically to analyse GSLT.

### A. Equation of State (EOS) of HDE

From conservation equation (2.6) of HDE, we get

$$\omega_D = -1 + (1+z) \frac{1}{3\omega_D} \frac{d\Omega_D}{dz} \quad (2.14)$$

Eliminating  $\frac{d\Omega_D}{dz}$  from eqs. (2.13) and (2.14) we get

$$\omega_D = -\frac{1}{3} - \frac{2}{3c} \frac{\sqrt{\Omega_D}}{\sqrt{\Omega_m + \Omega_D + \Omega_{r_c}} + \epsilon \sqrt{\Omega_{r_c}}} \quad (2.15)$$

As in GR based theory here also  $\omega_D < -1/3$

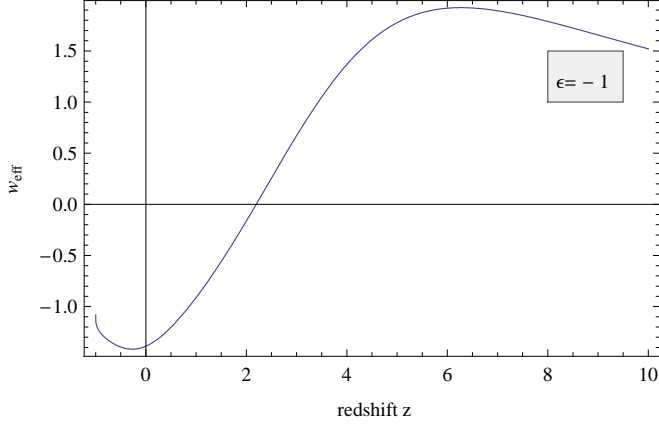


Fig.1

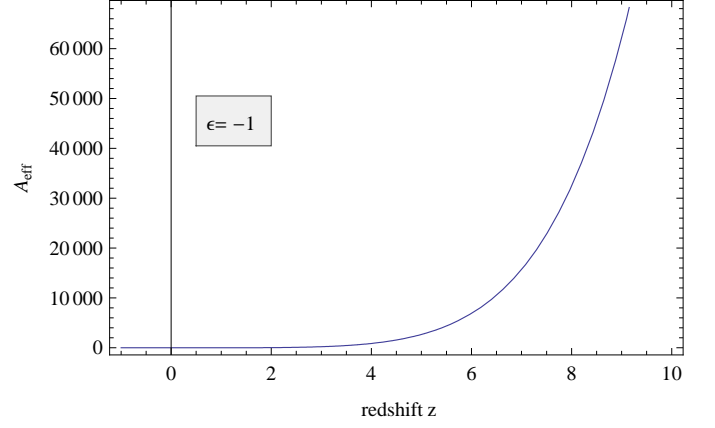


Fig.2

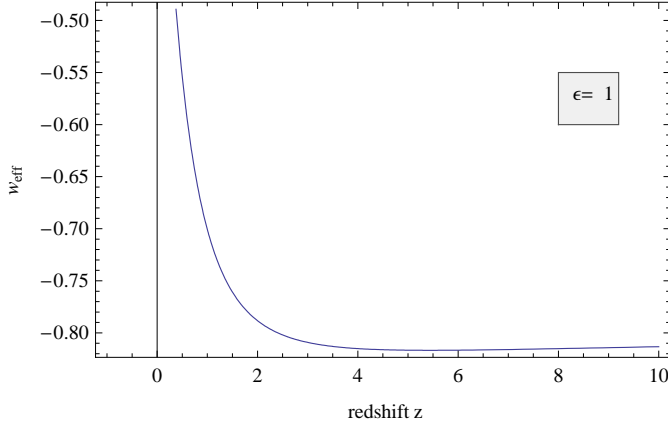


Fig.3

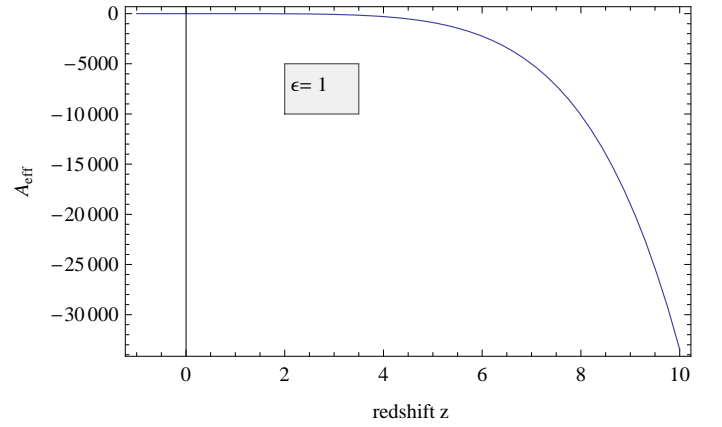


Fig.4

**Figs. 1 - 4** show the evolution of the  $\omega_{eff}$  and  $A_{eff}$  w.r.t  $z$ . The current density parameters used in the plots are  $\Omega_{m0} = 0.3$ ,  $\Omega_{r_c} = 0.12$ .

### B. EOS of effective dark energy

Defining  $\Omega_{eff} = \frac{\rho_{eff}}{3H_0^2}$ , from eq(2.3) we can write

$$\left(\frac{H}{H_0}\right)^2 = \Omega_m + \Omega_{eff} \quad (2.16)$$

Squaring eq (2.11) we have

$$\left(\frac{H}{H_0}\right)^2 = \Omega_m + \Omega_D + 2\Omega_{r_c} + 2\epsilon\sqrt{\Omega_{r_c}}\sqrt{\Omega_m + \Omega_D + \Omega_{r_c}} \quad (2.17)$$

Comparing eqs(2.16) and (2.17) we also find that

$$\Omega_{eff} = \Omega_D + 2\Omega_{r_c} + 2\epsilon\sqrt{\Omega_{r_c}}\sqrt{\Omega_m + \Omega_D + \Omega_{r_c}} \quad (2.18)$$

Now from conservation eq (2.7), we have

$$\Omega_{eff} = \Omega_{eff}^{(0)} \exp\left(3 \int_0^z \frac{1 + \omega_{eff}(z')}{1 + z'} dz'\right) \quad (2.19)$$

Equating eqs.(2.17) and (2.18) and then taking derivative on both sides w.r.t  $z$ , we get

$$1 + \omega_{eff} = \frac{1}{3\Omega_{eff}} \left[ \epsilon \sqrt{\Omega_{r_c}} \frac{3\Omega_m + (1+z) \frac{d\Omega_D}{dz}}{\sqrt{\Omega_m + \Omega_D + \Omega_{r_c}}} + (1+z) \frac{d\Omega_D}{dz} \right] \quad (2.20)$$

Now the above equation can be solved numerically with the help of differential equation (2.13) and (2.13a). The behavior of  $\omega_{eff}$  w.r.t  $z$  is shown in Figs. 1 and 3.

### III. THE GENERALIZED SECOND LAW OF THERMODYNAMICS:

In this section we examine the validity of GSL on 3-DGP brane. Let us consider a region of FRW universe enveloped by the horizon and assume that the region bounded by the horizon act as a thermal system with boundary defined by the horizon and is filled with a perfect fluid of energy density  $\rho_t$  and pressure  $p_t$ .

We take as in[15]

$$p_t = \omega_{eff} \rho_{eff} \quad \text{and} \quad \rho_t = \rho_m + \rho_{eff} \quad (3.1)$$

Gravity on the brane does not obey Einstein theory, therefore usual area formula for the black hole entropy may not hold on the brane. So we extract the entropy of the event horizon by assuming the first law of thermodynamics on event horizon [19].

The amount of energy crossing the horizon in time  $dt$  has the expression

$$-dE = 4\pi R_h^3 H (\rho_t + p_t) dt \quad (3.2)$$

where  $R_h$  is the radius of the horizon.

So assuming the first law of thermodynamics we have

$$\dot{S}_h = \frac{4\pi R_h^3 H}{T_h} \left[ \rho_m + (1 + \omega_{eff}) \rho_{eff} \right] \quad (3.3)$$

where  $S_h$  and  $T_h$  are the entropy and temperature of the horizon respectively. Using Gibb's equation [8],

$$T_h dS_I = dE_I + p_t dV$$

we obtain the variation of the entropy of the fluid inside the horizon as

$$\dot{S}_I = \frac{1}{T_h} \left[ V \dot{\rho}_t + (\rho_t + p_t) \dot{V} \right] \quad (3.4)$$

where  $S_I$  and  $E_I$  are the entropy and energy of the matter distribution inside the horizon. Here we assume as in ref [20] the temperature of the source inside the horizon is in equilibrium with the temperature associated with the horizon.

Connecting eqs. (2.5) and (2.7), we get

$$\dot{\rho}_t = -3H [\rho_m + (1 + \omega_{eff}) \rho_{eff}] \quad (3.5)$$

So starting with  $E_I = \frac{4}{3} \pi R_h^3 \rho_t$  and  $V = \frac{4}{3} \pi R_h^3$  and using eqs.(3.4) and (3.5) and after some simplification one gets

$$\dot{S}_I = \frac{4\pi R_h^2}{T_h} \left[ \rho_m + (1 + \omega_{eff}) \rho_{eff} \right] \left[ \dot{R}_h - H R_h \right] \quad (3.6)$$

Adding eqs. (3.3) and (3.6), one gets the resulting change of entropy

$$\dot{S}_{tot} = \dot{S}_h + \dot{S}_I = \frac{4\pi R_h^2}{T_h} \left[ \rho_m + (1 + \omega_{eff}) \rho_{eff} \right] \dot{R}_h \quad (3.7)$$

### A. The dynamical apparent horizon

Here we study the validity of GSLT on apparent horizon. The apparent horizon for flat space is defined as

$$R_A = \frac{1}{H}$$

In terms of apparent horizon Friedmann eq.(2.3) can be written as

$$\frac{1}{R_A^2} = \frac{1}{3}(\rho_m + \rho_{eff}) \quad (3.8)$$

If we take the derivative of eq.(3.8) w.r.t cosmic time, then we get

$$\dot{R}_A = \frac{HR_A^3}{2} [\rho_m + (1 + \omega_{eff})\rho_{eff}] \quad (3.9)$$

Substituting the above value of  $\dot{R}_A$  in eq.(3.7) we get

$$\dot{S}_{tot} = \dot{S}_A + \dot{S}_I = \frac{2\pi R_A^5 H}{T_A} [\rho_m + (1 + \omega_{eff})\rho_{eff}]^2 \geq 0 \quad (3.10)$$

It may be noted that the result is true irrespective of specific form of DE. Thus it supports earlier investigations in GR based different DE models like the generalized chaplygin gas [21], the HDE [8,22] etc.

### B. The cosmological event horizon

It is well known that in a spatial flat de Sitter universe the cosmological event horizon given by eq.(2.10) and the apparent horizon coincide and  $\dot{R}_E = 0$  [8]. Therefore for de Sitter space from eq.(3.7) we see that  $\dot{S}_{tot} = 0$ , which correspond to reversible adiabatic expansion.

Differentiating Friedmann eq.(2.3) w.r.t cosmic time, we get

$$\dot{H} = -\frac{1}{2} [\rho_m + (1 + \omega_{eff})\rho_{eff}] \quad (3.11)$$

Also from eq.(2.4), we have

$$\dot{\rho}_{eff} = \dot{\rho}_D + \epsilon \frac{3\dot{H}}{r_c} \quad (3.12)$$

Connecting this equation with conservation equation of effective energy density and eq (2.9), we get

$$\dot{R}_E = \frac{R_E^3}{2c^2} \left[ H(1 + \omega_{eff})\rho_{eff} - \frac{\epsilon}{2r_c} (\rho_m + (1 + \omega_{eff})\rho_{eff}) \right] \quad (3.13)$$

Connecting eqs (3.7) and (3.13), we get

$$\dot{S}_{tot} = \frac{2\pi R_E^5}{c^2 T_E} \left[ H(1 + \omega_{eff})\rho_{eff} - \frac{\epsilon}{2r_c} (\rho_m + (1 + \omega_{eff})\rho_{eff}) \right] [\rho_m + (1 + \omega_{eff})\rho_{eff}] \quad (3.14)$$

In terms of density parameters this equation can be rewritten as

$$\dot{S}_{tot} = \frac{18\pi H_0^5 R_E^5}{c^2 T_E} A_{eff}(z) \quad (3.15)$$

where  $A_{eff}(z)$  is defined as

$$A_{eff}(z) = \left[ \Omega_m + (1 + \omega_{eff})\Omega_{eff} \right] \left[ (\sqrt{\Omega_m + \Omega_D + \Omega_{r_c}} + \epsilon\sqrt{\Omega_{r_c}})(1 + \omega_{eff})\Omega_{eff} - \epsilon\sqrt{\Omega_{r_c}}(\Omega_m + (1 + \omega_{eff})\Omega_{eff}) \right] \quad (3.16)$$

Now  $\dot{S}_{tot} \geq 0$  if  $A_{eff}(z) \geq 0$ . After solving eqs (2.13,2.13a) together with eqs (2.18) and (2.20), the evolution  $A_{eff}(z)$  is plotted in figs 2 and 4.

The above results lead to the following conclusions :

**I.** For  $\epsilon = -1$ , the GSLT will be valid on the event horizon if  $(1 + \omega_{eff}) > 0$  i.e. effective DE is not of the phantom nature. Then universe as a thermodynamical system with two non-interacting fluid components (as in the present case) always obey the second law of thermodynamics.

Also for late time universe or DE dominated universe  $\Omega_m \rightarrow 0$  and in that case using eq.(2.11) we have

$$A_{eff}(z) = (1 + \omega_{eff})^2 \Omega_{eff}^2 \left[ \frac{H}{H_0} + \sqrt{\Omega_{r_c}} \right] \geq 0$$

Thus for DE dominated universe GSLT is always valid in this case.

Further one may note that, in this case the entropy of the event horizon also increases with time while variation of the matter entropy with time is not positive definite, but the sum of the entropies increases with the evolution of the universe. Also the radius of the event horizon increases with time.

**II.** For  $\epsilon = 1$  we see that  $A_{eff}(z) \geq 0$  if

$$\sqrt{\Omega_{r_c}} \leq \left[ \frac{\sqrt{\Omega_m + \Omega_D + \Omega_{r_c}} + \sqrt{\Omega_{r_c}}}{\Omega_m + (1 + \omega_{eff})\Omega_{eff}} \right] (1 + \omega_{eff})\Omega_{eff}$$

Thus in this case GSLT is valid subject to the above inequality.

For late time universe or DE dominated universe  $\Omega_m \rightarrow 0$  and using eq.(2.11) we have

$$A_{eff}(z) = (1 + \omega_{eff})^2 \Omega_{eff}^2 \left[ \frac{H}{H_0} - \sqrt{\Omega_{r_c}} \right]$$

Thus for DE dominated universe GSLT is valid in this case if  $H \geq H_0 \sqrt{\Omega_{r_c}}$

#### IV. CONCLUSIONS:

In this work, we have examined the validity of generalised second law of thermodynamics for universe as a thermodynamical system bounded by apparent horizon or event horizon in the DGP brane world scenario. Assuming the validity of the first law of thermodynamics, the GSLT is always satisfied on the apparent horizon irrespective of the equation of state for non interacting holographic dark energy and choice of  $\epsilon (= \pm 1)$ . For validity of GSLT on the event horizon, the choice of  $\epsilon$  as well as the equation of state for holographic dark energy is crucial. Figures 1 and 3 show the variation of  $\omega_{eff}$  as a function of redshift  $z$  for  $\epsilon = -1$  and  $+1$  respectively. For  $\epsilon = -1$ , the effective fluid may have phantom nature around  $z = 0$  while for  $\epsilon = +1$  the effective fluid can not have phantom behaviour throughout the evolution of the universe. Figures 2 and 4 show the validity of GSLT for  $\epsilon = -1$  and  $+1$  respectively. For  $\epsilon = -1$ , although  $A_{eff}$  is a decreasing function but it is positive throughout the evolution of the universe and hence GSLT is satisfied on the event horizon. However, for  $\epsilon = +1$ , although  $A_{eff}$  is an increasing function (decreasing function of  $z$ ) but it is negative throughout and hence there is a violation of GSLT.

Moreover, from observational data the estimate of the parameters are the following:  $\Omega_{m0} = 0.3$ ,  $\Omega_{r_c} = 0.12$ . Taking  $c = 1.2$ , for  $\epsilon = -1$ , using eqs.(2.13),(2.13a) and (2.20), we get  $w_{eff} = -1.38803$  at  $z = 0$  and for  $\epsilon = 1$  the value of  $w_{eff} = -0.157402$  at  $z = 0$ . Thus for  $\epsilon = -1$ , we have effective phantom behaviour and for  $\epsilon = 1$ , we have effective quintessence behaviour in the present universe.

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